# Problem A. Integers Exhibition

Input file: standard input
Output file: standard output

Time limit: 5 seconds Memory limit: 256 megabytes

Kazari loves math, she has been fascinated by divisors recently.

She defines a positive integer x K-magic if and only if there are more than K integers y where

- 0 < y < x.
- y has more divisors than x.

To show her research for those who loves math, she plans to hold an exhibition listing K-magic integers from small to large. Can you help her calculate the N-th integer in this exhibition?

### Input

The first line of the input contains an integer T ( $1 \le T \le 10^5$ ) denoting the number of test cases. Each test case consists of one line with two integers N, K ( $1 \le N \le 10^{18}, 0 \le K \le 233$ ).

## Output

For each test case, print an integer denoting the N-th K-magic integer.

standard input	standard output						
3	16						
8 0	387						
100 100	1507						
666 233							

# Problem B. Harvest of Apples

Input file: standard input
Output file: standard output

Time limit: 2 seconds Memory limit: 256 megabytes

There are n apples on a tree, numbered from 1 to n.

Count the number of ways to pick at most m apples.

#### Input

The first line of the input contains an integer T ( $1 \le T \le 10^5$ ) denoting the number of test cases. Each test case consists of one line with two integers n, m ( $1 \le m \le n \le 10^5$ ).

## Output

For each test case, print an integer representing the number of ways modulo  $10^9 + 7$ .

standard input	standard output						
2	16						
5 2	924129523						
1000 500							

## Problem C. Problems on a Tree

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 megabytes

Kazari loves to solve math problems.

In her opinion, all problems can be divided into three categories according to their difficulties - Easy, Medium and Hard, represented as 1, 2 and 3, respectively.

Today she goes to the forest and finds a strange tree with n vertices: there is a problem on each edge!

Formally, the *i*-th edge on the tree directly connects vertex  $u_i$  and  $v_i$  and contains a math problem which belongs to  $c_i$  category.

She makes a decision that she will **choose two endpoints** s, t **and walk from** t **to** s on each of next m days. During her walk, she must solve all problems that she will go through, but unfortunately, not on all days she will be able to do that - because the problems may be too hard!

More precisely, Kazari is able to solve all categories of problems at the beginning of a day, however, once she solve a Hard problem, she will lose all her faith at once, in the rest of the day she is only able to solve Easy problems, i.e., if she is encountered with a Medium or Hard problem during this time, she will fail.

There is a piece of good news that on each morning of next m days, exactly one problem will be chosen to become easier, i.e., a Hard problem will become a Medium problem, a Medium problem will become a Easy problem, a Easy problem will remain the same. Note that the effect is persistent.

Kazari would like to know, for each day, whether she is able to reach s from t and how many vertices from which she is able to reach s among all n vertices.

## Input

The first line of the input contains an integer T denoting the number of test cases.

Each test case starts with two integers n, m  $(1 \le n, m \le 10^5, \sum n, \sum m \le 3 \times 10^5)$  denoting the number of vertices and the number of days. Each of next n-1 lines contains three integers  $u_i, v_i, c_i$   $(1 \le u_i \ne v_i \le n, 1 \le c_i \le 3)$  describing an edge. Each of next m lines contains four integers  $a_i, b_i, s_i, t_i$   $(1 \le a_i \ne b_i \le n, 1 \le s_i, t_i \le n), (a_i, b_i)$  represents the edge where the problem is chosen to become easier and  $s_i, t_i$  represents the two endpoints that the girl chooses this day.

## Output

For each test case, print two integers for each day, the first integer denotes whether the girl is able to reach s and the second integer denotes the number of vertices from which she is able to reach s.

standard input	standard output
1	0 4
6 5	0 5
1 2 3	1 2
2 3 3	0 5
1 4 3	1 5
3 5 3	
2 6 3	
2 6 1 5	
3 5 2 4	
2 3 4 4	
2 3 2 4	
1 2 6 2	

# Problem D. Nothing is Impossible

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 megabytes

m students, including Kazari, will take an exam tomorrow.

The paper consists of exactly n problems, the i-th problem contains  $a_i$  correct answers and  $b_i$  incorrect answers, i.e. the i-th problem contains  $a_i + b_i$  candidates in total.

Each student should choose exactly one candidate as answer for each problem. If the answer to a certain problem is correct, then the student will get one point. The student who gets the most points wins.

Students only know the structure of the paper, but they are able to talk with each other during the exam. For the collective sense of honor, they intend to maximize the number of points that the winner will get. How many points the winner will get at least, if they take the optimal strategy?

#### Input

The first line of the input contains an integer T ( $1 \le T \le 100$ ) denoting the number of test cases.

Each test case starts with two integers n, m ( $1 \le n \le 100, 1 \le m \le 10^9$ ), denoting the number of problems and the number of students. Each of next n lines contains two integers  $a_i, b_i$  ( $1 \le a_i, b_i \le 100$ ), indicating the number of correct answers and the number of incorrect answers of the i-th problem.

## Output

For each test case, print an integer denoting the minimum number of points that the winner gets.

standard input	standard output						
2	1						
3 5	3						
1 3							
1 3							
1 3							
5 50							
1 1							
2 3							
3 2							
5 3							
3 5							

# Problem E. Matrix from Arrays

Input file: standard input
Output file: standard output

Time limit: 2 seconds Memory limit: 256 megabytes

Kazari has an array A length of L, she plans to generate an infinite matrix M using A.

The procedure is given below in C/C++:

```
int cursor = 0;
for (int i = 0; ; ++i) {
    for (int j = 0; j <= i; ++j) {
        M[j][i - j] = A[cursor];
        cursor = (cursor + 1) % L;
    }
}</pre>
```

Her friends don't believe that she has the ability to generate such a huge matrix, so they come up with a lot of queries about M, each of which focus the sum over some sub matrix. Kazari hates to spend time on these boring queries. She asks you, an excellent coder, to help her solve these queries.

## Input

The first line of the input contains an integer T ( $1 \le T \le 100$ ) denoting the number of test cases.

Each test case starts with an integer L ( $1 \le L \le 10$ ) denoting the length of A.

The second line contains L integers  $A_0, A_1, ..., A_{L-1}$   $(1 \le A_i \le 100)$ .

The third line contains an integer Q ( $1 \le Q \le 100$ ) denoting the number of queries.

Each of next Q lines consists of four integers  $x_0, y_0, x_1, y_1$  ( $0 \le x_0 \le x_1 \le 10^8, 0 \le y_0 \le y_1 \le 10^8$ ) querying the sum over the sub matrix whose upper-leftmost cell is  $(x_0, y_0)$  and lower-rightest cell is  $(x_1, y_1)$ .

#### Output

For each test case, print an integer representing the sum over the specific sub matrix for each query.

standard input	standard output					
1	1					
3	101					
1 10 100	1068					
5	2238					
3 3 3 3	33076541					
2 3 3 3						
2 3 5 8						
5 1 10 10						
9 99 999 1000						

## Problem F. Travel Through Time

Input file: standard input
Output file: standard output

Time limit: 5 seconds
Memory limit: 256 megabytes

Kazari, a girl who can travel through time, is playing chess on a number axis.

At the very beginning, there is a chess at position 0.

Then q events occurs in sequence, each of which belongs to one of the following five types:

- 1 x She places a chess at position x.
- 2 x She places a chess at position z if there exists chess at position y where  $|y-z| \leq x$ .
- 3 1 r She reverses [l, r], i.e., each chess at position x  $(l \le x \le r)$  moves to position r + l x.
- 4 x She travels to the time right after the x-th event.
- 5 x She checks if there exists chess at position x.

During the game, Kazari will tell you the q events in sequence. You are curious about the checking results of type-5 events, and decide to work out it timely according to the given information.

In order to keep your algorithm online, the input has been encrypted. You should XOR the current number of type-5 events whose result is Yes to each l, r and x, to get the real input.

#### Input

The first line of the input contains an integer T denoting the number of test cases.

Each test case starts with an integer q  $(1 \le q \le 50000, \sum q \le 10^6)$  denoting the number of events.

Each of next q lines contains an encrypted event, remember to decrypt it first!

- $0 \le x \le 10^6$  for type-2 events
- $|x| \le 10^{12}$  for type-1, type-5 events
- $|l|, |r| \le 10^{12}, l \le r$  for type-3 events
- It is guaranteed that x is less than the current number of events for type-4 events.

#### Output

For each test case, print Yes or No for each type-5 event.

standard input	standard output						
1	No						
9	Yes						
2 5	No						
5 6	Yes						
3 1 10	Yes						
5 6							
1 0							
5 2							
5 0							
4 0							
5 7							

## Problem G. Depth-First Search

Input file: standard input
Output file: standard output

Time limit: 5 seconds Memory limit: 256 megabytes

Kazari is learning depth-first search. More precisely, she is doing an experiment about it.

Consider an unrooted tree with n vertices and an empty array called A.

She randomly chooses a vertex s as root and starts from s to walk around, following the rules below.

- When she enters a vertex x for the first time, she append x to A at once.
- If some adjacent vertex has not been visited, she randomly chooses one and walks into it.
- If all adjacent vertices have been visited,
  - If she is at root, the experiment is done.
  - If she is not at root, she walks into the vertex which is the most nearest to root.

Among all possible arrays that A is likely to be finally, Kazari wishes to count how many of them is lexicographically smaller than the given array B. Since the answer is too large, print it modulo  $10^9 + 7$ .

## Input

The first line of the input contains an integer T denoting the number of test cases.

Each test case starts with a positive integer  $n \ (\sum n \le 10^6)$ , denoting the number of vertices.

The second line contains n integers  $B_1, B_2, ..., B_n$   $(1 \le B_i \le n, \forall i \ne j, B_i \ne B_j)$ .

Each of next n-1 lines contains two integers u, v, representing an edge (u, v) on the tree.

## Output

For each test case, print a non-negative integer denoting the answer modulo  $10^9 + 7$ .

standard input	standard output
2	3
5	9
2 1 3 5 4	
1 2	
2 3	
2 4	
4 5	
6	
6 4 5 3 2 1	
1 2	
2 3	
3 4	
4 5	
5 6	

## Problem H. Eat Cards, Have Fun

Input file: standard input
Output file: standard output

Time limit: 3 seconds
Memory limit: 256 megabytes

n cards with distinct integers  $a_i$  are arranged in a circle, numbered from 1 to n in clockwise order.

Initially, Kazari holds an empty array A and stands at card 1.

She will keep performing the following two operations until all cards have been eaten.

- She appends the number on the current card to A and eats the current card, with probability  $\frac{p}{a}$ .
- Move to the next card that has not been eaten in clockwise order.

It is obvious that A is a n-permutation at the end. Define its value as k if it is k-th lexicographically smallest among all n-permutations. Please help her work out the expected value of A.

#### Input

The first line of the input contains an integer T denoting the number of test cases.

Each test case starts with a positive integer n ( $n \le 300, \sum n \le 1500$ ), denoting the number of cards.

The second line contains two integers p, q (0 ), representing the probability.

The third line contains n integers, the i-th integer represents the number  $a_i$   $(1 \le a_i \le n)$  on the i-th card.

## Output

For each test case, print the expected value E modulo  $10^9 + 7$ .

Let E be an irreducible fraction  $\frac{a}{b}$ , define  $E \equiv a\hat{b} \pmod{p}$ , where  $b\hat{b} \equiv 1 \pmod{p}$ , assuming p is a prime.

standard input	standard output					
2	33333337					
2	841859064					
1 2						
1 2						
5						
3 11						
5 2 3 4 1						

# Problem I. Delightful Formulas

Input file: standard input
Output file: standard output

Time limit: 10 seconds Memory limit: 256 megabytes

Kazari is too bored this summer holiday, she decides to play with formulas.

First of all, she comes up with two positive integers N, K.

She loves power, therefore she builds a sequence a where  $a_i = i^K$   $(1 \le i \le N)$ .

She loves summation, therefore she calculate the partial sum of a as s, i.e.  $s_i = \sum_{j \le i} a_j$   $(1 \le i \le N)$ .

She also loves coprimes, therefore she calculate the sum of elements in s whose index i is coprime to N, i.e.  $v = \sum_{1 \le i \le N, \gcd(i,N)=1} s_i$ 

Your task is to work out v. Since the answer is too large, print it modulo 998244353.

## Input

The first line of the input contains an integer T denoting the number of test cases.

Each test case starts with a positive integer K  $(K \le 10^5, \sum K \le 10^6)$ .

The second line contains a positive integer m ( $m \le 20$ ), indicating the number of distinct primes of N.

Each of next m lines contains two positive integers  $p_i$ ,  $a_i$  where  $p_i$  is a prime  $(p_i, a_i \le 10^9, p_i < p_{i+1})$ .

$$N = \prod_{i=1}^{m} p_i^{a_i}$$

## Output

For each test case, print an integer representing the answer modulo 998244353.

standard input	standard output					
2	16					
1	32727388					
2						
2 1						
3 1						
233						
1						
23333 1						

## Problem J. Let Sudoku Rotate

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 megabytes

Sudoku is a logic-based, combinatorial number-placement puzzle, which is popular around the world.

In this problem, let us focus on puzzles with  $16 \times 16$  grids, which consist of  $4 \times 4$  regions. The objective is to fill the whole grid with hexadecimal digits, i.e. 0123456789ABCDEF, so that each column, each row, and each region contains all hexadecimal digits. The figure below shows a solved sudoku.

6	8	1	D	5	Α	0	С	4	3	Е	9	В	2	F	7
0	Α	7	3	4	В	6	2	8	С	1	F	D	9	Ε	5
5	С	9	В	7	3	Е	F	А	2	6	D	8	4	1	0
F	2	4	Е	D	1	8	9	5	0	7	В	C	Α	6	3
3	9	F	Α	Ε	D	5	6	2	8	С	1	0	В	7	4
D	1	2	0	С	4	В	7	9	5	Α	6	Е	F	3	8
7	Е	С	8	2	9	Α	0	F	В	3	4	6	D	5	1
В	5	6	4	3	8	F	1	7	Е	D	0	9	С	2	Α
2	6	Α	5	В	С	4	Е	3	D	0	8	7	1	9	F
9	0	Е	С	1	6	7	3	В	F	4	2	Α	5	8	D
4	D	8	7	Α	F	2	5	6	1	9	С	3	0	В	Ε
1	3	В	F	9	0	D	8	Е	Α	5	7	2	6	4	С
С	7	D	1	F	2	9	4	0	6	8	3	5	Ε	Α	В
Α	F	5	2	8	7	С	D	1	9	В	Ε	4	3	0	6
8	В	3	6	0	Ε	1	Α	D	4	2	5	F	7	С	9
Ε	4	0	9	6	5	3	В	С	7	F	Α	1	8	D	2

Yesterday, Kazari solved a sudoku and left it on the desk. However, Minato played a joke with her - he performed the following operation several times.

• Choose a region and rotate it by 90 degrees counterclockwise.

She burst into tears as soon as she found the sudoku was broken because of rotations.

Could you let her know how many operations her brother performed at least?

#### Input

The first line of the input contains an integer T ( $1 \le T \le 10^3$ ) denoting the number of test cases. Each test case consists of exactly 16 lines with 16 characters each, describing a broken sudoku.

## Output

For each test case, print a non-negative integer indicating the minimum possible number of operations.

# Example

standard input	standard output
1	5
681D5A0C9FDBB2F7	
0A734B62E167D9E5	
5C9B73EF3C208410	
F24ED18948A5CA63	
39FAED5616400B74	
D120C4B7CA3DEF38	
7EC829A085BE6D51	
B56438F129F79C2A	
5C7FBC4E3D08719F	
AE8B1673BF42A58D	
60D3AF25619C30BE	
294190D8EA57264C	
C7D1B35606835EAB	
AF52A1E019BE4306	
8B36DC78D425F7C9	
E409492FC7FA18D2	

## Note

The original sudoku is same as the example in the statement.

6	8	1	D	5	Α	0	С	4	3	E	9	В	2	F	7
0	Α	7	3	4	В	6	2	8	С	1	F	D	9	Е	5
5	С	9	В	7	3	Е	F	Α	2	6	D	8	4	1	0
F	2	4	Ε	D	1	8	9	5	0	7	В	С	Α	6	3
3	9	F	Α	Ε	D	5	6	2	8	C	1	0	В	7	4
D	1	2	0	С	4	В	7	9	5	A	6	Ε	F	3	8
7	Е	С	8	2	9	Α	0	F	В	3	4	6	D	5	1
В	5	6	4	3	8	F	1	7	E	D	0	9	С	2	Α
2	6	A	5	В	С	4	E	3	D	0	8	7	1	9	F
9	0	E	С	1	6	7	3	В	F	4	2	Α	5	8	D
4	D	8	7	А	F	2	5	6	1	9	С	3	0	В	Е
1	3	В	F	9	0	D	8	Ε	Α	5	7	2	6	4	С
С	7	D	1	F	2	9	4	0	6	8	3	5	Е	Α	В
Α	F	5	2	8	7	5	D	1	9	В	Ε	4	3	0	6
8	В	3	6	0	E	1	A	D	4	2	5	F	7	С	9
Ε	4	0	9	6	5	3	В	С	7	F	Α	1	8	D	2

# Problem K. Expression in Memories

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 megabytes

Kazari remembered that she had an **expression**  $s_0$  before.

Definition of **expression** is given below in Backus–Naur form.

- <expression> ::= <number> | <expression> <operator> <number>
- <operator> ::= "+" | "\*"
- <number> ::= "0" | <non-zero-digit> <digits>
- <digits> ::= "" | <digits> <digit>
- <digit> ::= "0" | <non-zero-digit>
- <non-zero-digit> ::= "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9"

For example, 1\*1+1, 0+8+17 are valid expressions, while +1+1, +1\*+1, 01+001 are not.

Though  $s_0$  has been lost in the past few years, it is still in her memories.

She remembers several corresponding characters while others are represented as question marks.

Could you help Kazari to find a possible valid expression  $s_0$  according to her memories, represented as s, by replacing each question mark in s with a character in 0123456789+\*?

#### Input

The first line of the input contains an integer T denoting the number of test cases.

Each test case consists of one line with a string s  $(1 \le |s| \le 500, \sum |s| \le 10^5)$ .

It is guaranteed that each character of s will be in 0123456789+\*? .

#### Output

For each test case, print a string  $s_0$  representing a possible valid expression.

If there are multiple answers, print any of them.

If it is impossible to find such an expression, print IMPOSSIBLE.

standard input	standard output
5	11111
?????	0+0+0
0+0+0	IMPOSSIBLE
?+*??	10+10
?0+?0	IMPOSSIBLE
?0+0?	

# Problem L. Graph Theory Homework

Input file: standard input
Output file: standard output

Time limit: 1 second Memory limit: 256 megabytes

There is a complete graph containing n vertices, the weight of the i-th vertex is  $w_i$ .

The length of edge between vertex i and j  $(i \neq j)$  is  $\lfloor \sqrt{|w_i - w_j|} \rfloor$ .

Calculate the length of the shortest path from 1 to n.

## Input

The first line of the input contains an integer T ( $1 \le T \le 10$ ) denoting the number of test cases. Each test case starts with an integer n ( $1 \le n \le 10^5$ ) denoting the number of vertices in the graph.

The second line contains n integers, the i-th integer denotes  $w_i$  ( $1 \le w_i \le 10^5$ ).

## Output

For each test case, print an integer denoting the length of the shortest path from 1 to n.

standard input	standard output
1	2
3	
1 3 5	