牛客网暑期ACM多校训练营（第七场）

一. 编程题

1. You have a complete bipartite graph where each part contains exactly n nodes, numbered from 0 to n - 1 inclusive.

The weight of the edge connecting two vertices with numbers x and y is $x \wedge y$ (bitwise AND).

Your task is to find a minimum cost perfect matching of the graph, i.e. each vertex on the left side matches with exactly one vertex on the right side and vice versa. The cost of a matching is the sum of cost of the edges in the matching.

$\wedge$ denotes the bitwise AND operator. If you're not familiar with it, see {https://en.wikipedia.org/wiki/Bitwise_operation#AND}.

输入描述：
The input contains a single integer n ($1 \le n \le 5 * 10^5$).
输出描述：
Output n space-separated integers, where the i-th integer denotes p_i ($0 \le p_i \le n - 1$, the number of the vertex in the right part that is matched with the vertex numbered i in the left part. All p_i should be distinct.

Your answer is correct if and only if it is a perfect matching of the graph with minimal cost. If there are multiple solutions, you may output any of them.
示例1:
输入
3
输出
0 2 1
说明
For n = 3, $p_0 = 0$, $p_1 = 2$, $p_2 = 1$ works. You can check that the total cost of this matching is 0, which is obviously minimal.

正确答案：

2. In a strange kingdom, there are N months in one year and there are M days in one month. A day in a year of the Kingdom can be represented by a pair (x, y) where $1 \le x \le N$ denotes the month and $1 \le y \le M$ denotes the day.

A nonempty set of days S is given. Charlotte's birthday is one of the elements of S. Alice, Bob and David know this fact but they didn't know any other information about her birthday. Let's denote Charlotte's birthday as (c_x, c_y).

Charlotte tells Alice the value of c_x and tells Bob the value of c_y.
After that, the following exchange happened exactly L times :

Alice says that she does not know Charlotte's birthday after hearing the last statement (if any) and Bob says that he does not know Charlotte's birthday after Alice's last statement.

After the above happens L times, Alice says that she knows Charlotte's birthday (i.e. she figured out the value of c_y). After hearing that, Bob says that he knows Charlotte's birthday as well (i.e. he figured out the value of c_x. Note that he might also have figured out Charlotte's birthday before Alice's statement). Finally, after hearing both statements, David says that he knows Charlotte's Birthday (i.e. he figured out both values c_x, c_y, possibly before some statements are made).

Assume that all the people are perfect logicians and do not lie.

Among the 2^{NM} - 1 possible nonempty sets of days S, how many of them are there such that the scenario above is possible, modulo 10^9 + 7?

输入描述：
The input contains three space-separated integers, N, M, L (1 ≤ N, M, L ≤ 30).
输出描述：
Output a single integer, the answer to the problem.
示例1:
输入
3 3 1
输出
18

正确答案：

3. A binary string s of length N = 2^n is given. You will perform the following operation n times :

- Choose one of the operators AND (&), OR (|) or XOR (^). Suppose the current string is S = $s_1s_2...s_k$. Then, for all $1 \leq i \leq \frac{k}{2}$, replace $s_{2i-1}s_{2i}$ with the result obtained by applying the operator to s_{2i-1} and s_{2i}. For example, if we apply XOR to {1101} we get {01}.

After n operations, the string will have length 1.

There are 3^n ways to choose the n operations in total. How many of these ways will give 1 as the only character of the final string.

输入描述：
The first line of input contains a single integer n (1 ≤ n ≤ 18).

The next line of input contains a single binary string s (|s| = 2^n). All characters of s are either 0 or 1.
输出描述：
Output a single integer, the answer to the problem.
示例1:
输入
2
1001

输出
4
说明
The sequences (XOR, OR), (XOR, AND), (OR, OR), (OR, AND) works.

正确答案：

4. Here's a simple problem :

"Given integers A, B, X, N and a prime P, let f(x) = Ax + B and define $f^n(x)$ as f(f(...f(x)...)), where the function f is iterated n times. Calculate the value of $f^N(X)$ modulo P."

This is a classic problem, but have you ever tried the inverse problem?

"Given x, n, t, p, where 0 ≤ t < p ≤ n and p is a prime. Find nonnegative integers a, b where 1 ≤ a ≤ p - 1, 0 ≤ b ≤ p - 1 such that the answer to the testcase (A, B, X, N, P) = (a, b, x, n, p) is t, or determine if it's impossible."

This is still too easy, so now let's try to solve the inverse of the inverse problem.

For a testcase x, n, t, p to the inverse problem, define g(x, n, t, p) as -1 if there is no solution for a, b and the smallest possible value of a in a valid solution to this testcase otherwise. For example, g(1, 2, 31, 101) = 1 as a = 1, b = 15 is a valid solution.

Given an integer M, find a testcase x, n, t, p such that p ≤ M and the value of g(x, n, t, p) is maximal.

输入描述：
The input contains a single integer, M (3 ≤ M ≤ 10^5).
输出描述：
The output should contain 4 space-separated integers, x, n, t, p, denoting your testcase. The testcase must satisfy the constraints 1 ≤ x ≤ 10^9, 1 ≤ n ≤ 10^{18}, 0 ≤ t < p ≤ M and p is prime.

Your testcase will be accepted if it satisfies all constraints and g(x, n, t, p) is maximum possible under the current constraints. If there are multiple solutions, you may output any of them.
示例1:
输入
8
输出
2018 231 1 7
说明
You can check that g(2018, 231, 1, 7) = 3. (For instance, a = 3, b = 4 is a solution and there are no solutions for a ≤ 2)

正确答案：

5. You love doing graph theory problems. You've recently stumbled upon a classical problem : Count the number of 4-cliques in an undirected graph.

Given an undirected simple graph G, a 4-clique of G is a set of 4 nodes such that all pairs of nodes in this set are directly connected by an edge.

This task would be too easy for you, wouldn't it? Thus, your task here is to find an undirected simple graph G with exactly k 4-cliques. Can you solve this task?

输入描述：

The first line of input contains a single integer k ($1 \le k \le 10^6$).

输出描述：

On the first line, output two space-separated integers, $n, m (1 \leq n \leq 75, 1 \leq m \leq \frac{n(n-1)}{2})$. On the next m lines, output two space-separated integers denoting an edge of the graph u, v (1 ≤ u, v ≤ n), where u and v are the endpoints of the edge.

Your graph must not contain any self-loops or multiple edges between the same pair of nodes. Any graph that has exactly k 4-cliques and satisfies the constraints will be accepted. It can be proven that a solution always exist under the given constraints.

示例1:

输入

1

输出

4 6
1 2
1 3
1 4
2 3
2 4
4 3

说明

In the sample, the whole graph is a 4-clique.

正确答案：

6. Given a set X of distinct positive integers, denote mindiff(X) as the minimum absolute difference of two distinct elements in X and maxdiff(X) as the maximum absolute difference of two distinct elements in X. If the size of X is strictly less than 2, define mindiff(X) and maxdiff(X) as 0.

Find the sum of mindiff$(T)\cdot$maxdiff(T), where T varies over all possible subsets of S = {1, 2, ..., n}, modulo $10^9 + 7$.

输入描述：

The input contains a single integer, n ($1 \le n \le 5 * 10^5$).

输出描述：

Output a single integer, the answer to the problem.

示例1:

输入

3

输出

8
说明
For sample 1, the subsets with size ≤ 1 will not contribute to the answer.

mindiff$(\{1,2\})\cdot$maxdiff$(\{1,2\}) = 1 \cdot 1 = 1$

mindiff$(\{2,3\})\cdot$maxdiff$(\{2,3\}) = 1 \cdot 1 = 1$

mindiff$(\{1,3\})\cdot$maxdiff$(\{1,3\}) = 2 \cdot 2 = 4$

mindiff$(\{1,2,3\})\cdot$maxdiff$(\{1,2,3\}) = 1 \cdot 2 = 2$

The total sum is 8.
示例2:
输入
5
输出
106

正确答案：

7. You have found an ancient record about a rock-paper-scissors tournament. There are N = 2^n players in the tournament and the tournament is played in a single-elimination format. The players are numbered from 1 to N inclusive. There are at most 32 players in the tournament.

The tournament consists of n - 1 rounds. In the first round, player 2i - 1 will play against player 2i for all 1 ≤ i ≤ 2^{n-1}. The loser of each round is eliminated. The process repeats with the winner of the game between players 2i - 1 and 2i relabelled as player i until only one participant remain. That participant is the winner of the tournament.

In ancient times, the people are not very clever. You know from your history textbook that there are 3 types of people : one who always plays Rock, one who always plays Scissors, and one who always plays Paper. We denote them by R, S, P respectively for convenience.

In the record, it is stated that participant i is of type s_i. Unfortunately, some records are not clear and thus you can't determine which type some participants are. Thus, you replace their type with a question mark instead.

Therefore, the record looks like a string of N characters, s = $s_1s_2...s_N$, where s_i denotes the type of participant i, or is equal to '?' (without quotes) if the type is unknown.

After that, you calculated the number of ways A, B, C the question marks can be replaced with one of the characters R, S or P so that the winner of the tournament is of type R, S, P respectively. (Note that paper beats rock, rock beats scissors and scissors beats paper)

The next day, you realized that the paper containing the string s is gone. However, you still remember the values of A, B, C. Can you find a string s with length ≤ 32 such that the number of ways to fill in the question marks with one of the letters R, S or P so that a player of type R, S and P wins the tournament is A, B, C respectively?

输入描述：
The first line of input contains a single integer T (1 ≤ T ≤ 50).

The next T lines contain three space-separated integers each, A, B, C ($1 \le A, B, C \le 3^{32}$).
输出描述：
For each testcase, output a single line consisting of a string s that satisfies the conditions. The length of s must be a power of 2 between 1 and 32 inclusive. Each character of s should be either 'R', 'S', 'P' or '?' (without quotes). If there are multiple solutions, you may output any of them. If no such string satisfying the conditions exist, output a single line consisting of the word "Impossible" (without quotes, case-sensitive) instead.
示例1:
输入
3
1 2 6
3 5 7
1 1 1
输出
RR?P?PRP
Impossible
?
说明
For the first testcase, the only way for a R-type guy to win the tournament is when the question marks are filled in this way : {RRSPSPRP}

After the first round, the remaining participants are {RSSP}.

After the second round, the remaining participants are {RS}.

After the last round, the remaining participant is {R}, as desired.

There are 2 and 6 ways to fill in the question marks for a S-type and P-type player to win respectively.

正确答案：

8. There are n students in a school. The final exams are over and the results are out. Student i scored s_i points in the final exam.

You are the principal of the school and you want to divide the students into m classes. The i-th class should contain at least l_i students and at most r_i students. Each student must belong to exactly one class.

The score of a class is defined as the average scores of all the students in the class. The score of the school is the sum of the scores of all classes. Your task is to find the maximum possible score of the school.

输入描述：
The first line contains a single integer n (1 ≤ n ≤ 250), denoting the number of students.

The next line contains n space-separated integers, $s_1, s_2, \ldots, s_n$ ($1 \le s_i \le 10^6$).

The next line contains a single integer m (1 ≤ m ≤ 30), denoting the number of classes.

The next m lines contains two space-separated integers each, l_i, r_i, describing the i-th class.

It is guaranteed that the input is chosen such that there exist a way to assign all the students into classes.
输出描述：
Print the only number — the answer for the problem. Your answer is considered correct if its absolute or relative error does not exceed 10^{-6}.

Formally, let your answer be a, and the jury's answer be b. Your answer is considered correct if
$$\frac{|a-b|}{\max(1,|b|)} \le 10^{-6}$$
.
示例1:
输入
5
3 6 1 2 5
3
1 3
1 1
2 3
输出
13.0000000000
说明
The optimal strategy is to assign the students with scores 1, 2, 3 into the third class, assign the student with score 5 into the first class and assign the student with score 6 into the second class. Then, the scores of the classes are $5, 6, \frac{1+2+3}{3}=2$ respectively. The score of the school is 5 + 6 + 2 = 13.

正确答案：

9. You are given an undirected, unweighted tree T on n nodes. We define the distance between two nodes u, v as the number of edges we need to travel from u to v. We define the diameter of a subset of nodes S as the maximum distance between two elements in S. If the size of S is smaller than or equal to 1, we define the diameter as 0.

Find the number of subsets S of nodes in T such that the diameter of S is exactly D.

输入描述：
The first line of input contains a single integer n (2 ≤ n ≤ 100000).

The next n - 1 lines contains two space-separated integers each, u_i, v_i ($1 \le u_i \ne v_i \le n$), describing an edge of the tree connecting u_i and v_i.

The last line contains a single integer D (1 ≤ D ≤ n).

It is guaranteed that the input graph is a tree.
输出描述：
Output a single integer, the number of subsets of nodes with diameter D. Since the answer might be large,

output it modulo $10^9 + 7$.
示例1:
输入
4
2 3
1 3
3 4
2
输出
8
说明
For the first sample, the subsets are {1, 2}, {1, 4}, {2, 4}, {1, 2, 3}, {1, 3, 4}, {2, 3, 4}, {1, 2, 4}, {1, 2, 3, 4}.
示例2:
输入
4
2 3
1 3
3 4
1
输出
3
说明
For the second sample, the subsets are {1, 3}, {2, 3}, {3, 4}.

正确答案：

10. You have a n * m grid of characters, where each character is an English letter (lowercase or uppercase, which means there are a total of 52 different possible letters).

A nonempty subrectangle of the grid is calledsudoku-likeif for any row or column in the subrectangle, all the cells in it have distinct characters.

How manysudoku-likesubrectangles of the grid are there?

输入描述：
The first line of input contains two space-separated integers n, m (1 ≤ n, m ≤ 1000).

The next n lines contain m characters each, denoting the characters of the grid. Each character is an English letter (which can be either uppercase or lowercase).
输出描述：
Output a single integer, the number of **sudoku-like** subrectangles.
示例1:
输入
2 3
AaA
caa
输出
11

说明
For simplicity, denote the j-th character on the i-th row as (i, j).

For sample 1, there are 11 **sudoku-like** subrectangles. Denote a subrectangle by (x_1, y_1, x_2, y_2), where (x_1, y_1) and (x_2, y_2) are the upper-left and lower-right coordinates of the subrectangle.

The **sudoku-like** subrectangles are (1, 1, 1, 1), (1, 2, 1, 2), (1, 3, 1, 3), (2, 1, 2, 1), (2, 2, 2, 2), (2, 3, 2, 3), (1, 1, 1, 2), (1, 2, 1, 3), (2, 1, 2, 2), (1, 1, 2, 1), (1, 3, 2, 3).
示例2:
输入
4 5
abcde
fGhij
klmno
pqrst
输出
150
说明
For sample 2, the grid has 150 nonempty subrectangles, and all of them are **sudoku-like**.

正确答案：